9. Yu. A. Burenin and G. A. Shvetsov, "The possibilities of an explosive MHD generator as a source of energy for a plasma focus," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1981).
10. N. P. Kozlov and Yu. S. Protasov, "Experimental investigation of a plasma focus in erosion-plasma accelerators," Zh. Tekh. Fiz., 52, No. 8 (1982).
11. A. S. Kamrukov, N. P. Kozlov, and Yu. S. Protasov, "Dynamics and radiation of open (vacuum) plasma-dynamic discharges of the 'plasma-focus' type," Teplofiz. Vys. Temp., 20, No. 2 (1982).
12. L. A. Leskov, M. Z. Mukhoyan, and V. V. Savichev, "Investigation of a pulsed electromagnetic source of erosion plasma," in: Problems of Low-Temperature Plasma Physics [in Russian], Nauka i Tekhnika, Minsk (1970).
13. V. V. Vladimirov et al., "A magnetoplasma compressor with an explosive magnetic energy generator," Zh. Tekh. Fiz., 50, No. 7 (1980).

STATIONARY DISCHARGE ACCOMPANYING EMERGENCE OF THE MAGNETIC FLUX THROUGH THE SURFACE OF AN INSULATOR

S. F. Garanin, E. S. Pavlovskii, and V. B. Yakubov

UDC 537.52

It is shown in [1, 2] that when a magnetic flux flows through the surface of an insulator, a stationary surface discharge, which limits the velocity of outflow of the magnetic force lines, can appear. A theory of such a discharge, using a number of simplifying assumptions, in particular the assumption of total single-ionization of the vapor of the insulator flowing out of the discharge, which is valid for not very strong magnetic fields $H \sim 10^4$ Oe, is developed in [2]. In this paper we examine the more general case of arbitrary multiple ionization, which is important for stronger magnetic fields, in particular fields used in experiments on magnetic acceleration of shells (see, for example, [3]).

In the problem under examination the mutually perpendicular magnetic H and electric E fields are parallel to the surface of the insulator, which we assume is flat. The self-sustaining surface discharge along the vapor of the insulator is realized due to the fact that the outflow of plasma driven by the ponderomotive force from the surface is compensated by vaporization of new sections of the insulator by the thermal radiation from the plasma being carried away. The ionized vapor entering into the discharge zone continues to be heated by Joule heat and is accelerated until the plasma velocity reaches the velocity v_1 of the outflow of magnetic force lines and the electric field in the comoving coordinate system vanishes.

Under typical experimental conditions, the thickness of the discharge x_H is much smaller than the dimensions L of the region in which the vapor of the insulator moves (for $H \sim 10^4$ Oe, x_H is of the order of 0.1 cm and decreases with increasing H). For this reason, the time for restructuring of the vaporization regime is much shorter than the characteristic time for the change in the magnetic fields or other quantities that affect the current layer, and the discharge may be assumed to be stationary. In solving the complete magnetohydrodynamic problem, which describes the action of one or another experimental setup, in which such a discharge occurs, the discharge zone can be replaced by an infinitely narrow jump in all MHD quantities. The purpose of this work is to obtain the conditions on this jump; for this it is necessary to find the dependence of the velocity v_1 of outflow of the plasma as well as its density and temperature on the magnitudes of the magnetic fields in the unvaporized insulator H_0 and at the outlet from the current layer H_1.

A significant factor is that, in order to obtain the dependences indicated, strictly speaking, it is not sufficient to know only the integral laws of conservation, relating the quantities at the inlet and outlet of the discharge zone; it is also necessary to solve the problem of the structure of this zone. In [2] it was possible to circumvent the solution of this more complicated problem only by making an approximation in which the temperature of

---

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 9-15, March-April, 1984. Original article submitted January 26, 1983.

0021-8944/84/2502-0171$08.50 © 1984 Plenum Publishing Corporation

the plasma flowing out of the discharge is assumed to be high enough for practically complete first ionization, but simultaneously low enough so that further ionization can be neglected; in addition, the thermal energy is assumed to be much smaller than the ionization energy, and the thermal pressure is assumed to be much smaller than the magnetic pressure. In this work the formulation of the problem is free of the quite rigid restrictions indicated above [2]. From the mathematical point of view, the problem is an eigenvalue problem.

The dependence of all quantities on the normal coordinate x is described by the system of stationary MHD equations:

$$\rho v = \text{const}; \tag{1}$$

$$P + \rho v^2 + H^2/8\pi = \text{const}; \tag{2}$$

$$\rho v(w + v^2/2) - Q - cHE/4\pi = \text{const}; \tag{3}$$

$$-\varkappa dH/dx + vH = cE = \text{const}, \tag{4}$$

where $\rho(x)$, $v(x)$, $P(x)$, $H(x)$, $w(x)$, $Q(x)$, and $\varkappa(x)$ are the instantaneous values of the density, velocity, pressure, magnetic field, specific enthalpy, heat flux, and coefficient of magnetic diffusion, respectively; E is the electric field; and c is the velocity of light. The system (1)-(4) represents the laws of conservation of mass, momentum, energy, and magnetic flux in a coordinate system in which the current layer is at rest. It may be assumed that this system coincides with the laboratory system, since the characteristic density in the current layer is much smaller than the density of the insulator.

The solution of the problem is simplified by the fact that in cases of practical interest the analysis of the heat flux Q can be greatly simplified. We shall estimate the optical thickness of the current layer x_H/λ (λ is the Rosselund mean free path of the radiation, $x_H \sim \varkappa/v$), starting from the diffusion approximation in the description of the heat transfer. Within the framework of this approximation, the coefficient of thermal diffusivity $\sim\lambda\sigma T^4/\rho w$ and the magnetic diffusion $\varkappa$ must be of the same order of magnitude (here, σ is the Stefan–Boltzmann constant, T is the temperature). Using also the relation

$$\rho w \sim \rho v^2 \sim H^2/8\pi$$

and power-law dependences of P, λ, and $\varkappa$ on ρ and T, we find

$$x_H/\lambda \sim \varkappa_H/\lambda v \sim \sigma T^4/(\rho w v) \sim 0.2H^{0.14},$$

where H is measured in MOe.

Thus, for a typical insulator containing light elements and not very high magnetic fields, the optical thickness of the current layer is small. For this reason, the heat flux Q toward the insulator from the plasma leaving the current layer and having a temperature T_1 must be assumed to equal $\sigma T^4{}_1$ and, in addition, the quantity Q remains practically unchanged over the thickness x_H, where the remaining quantities (H, ρ, v, T, and the fluxes of gasdynamic and magnetic energy) change considerably. Absorption of thermal flux begins at distances $x \sim \lambda$, i.e., at the outlet from the current layer toward the insulator, where the changes in the remaining fluxes in Eq. (3) are much smaller than their characteristic values in the current layer. Therefore, the entire region is separated into two zones: zone I of the current layer, where the quantity Q in (3) can be neglected, and zone II of absorption of thermal flux, where in (3), aside from Q, only terms which are first-order infinitesimal with respect to the velocity v need be retained.

The indices 0 denote quantities in the initial section of the discharge (on the side of the unvaporized insulator); the indices 1 indicate quantities at the end of the discharge zone. Since the density of the insulator is high, $v_0 = 0$. The heat flux does not penetrate deep into the insulator, so that $Q_0 = 0$. The current density equals zero at the outlet from the current layer, so that from (4) it follows that

$$v_1 H_1 = cE. \tag{5}$$

We introduce the dimensionless variables

$$u \equiv v/v_1, \quad h \equiv H/H_1, \quad q \equiv Q/v_1 \cdot (H_1^2/8\pi), \quad p = P/P_1,$$

as well as the parameters μ and ν

$$\mu \equiv \rho_1 v_1^2/(H_1^2/8\pi), \quad \nu \equiv P_1/(H_1^2/8\pi).$$

Then, treating the insulator vapor as a gas with an adiabatic index γ and using (1)-(5), we rewrite Eqs. (2) and (3) in the dimensionless form

$$\nu p + \mu u + h^2 = \nu p_0 + h_0^2 = \nu + \mu + 1; \tag{6}$$

$$\frac{\gamma}{\gamma-1}\nu p u + \mu u^2/2 - q + 2h = 2h_0 = \frac{\gamma}{\gamma-1}\nu + \frac{\mu}{2} - q_1 + 2. \tag{7}$$

From Eqs. (6) and (7) we can derive a relationship between μ and ν, using the smallness of q in zone I. Neglecting q and eliminating p from (6) and (7), we obtain

$$u = \left\{\frac{\gamma}{\gamma-1}(\nu + \mu + 1 - h^2) \pm \sqrt{D}\right\} \Big/ \frac{\gamma+1}{\gamma-1}\mu, \tag{8}$$

where

$$D = \left[\frac{\gamma}{\gamma-1}(\nu + \mu + 1 - h^2)\right]^2 - \frac{2(\gamma+1)}{\gamma-1}\mu\left(2 + \frac{\mu}{2} + \frac{\gamma}{\gamma-1}\nu - 2h\right). \tag{9}$$

Analysis of formulas (8) and (9), using the conditions u = 1 at h = 1 and u = 0 at h = h_0 > 1, shows that at h = 1 the plus sign must be used in the radical in (8). On the other hand, at the point h ≃ h_0, u ≃ 0 at the boundary of zones I and II, the minus sign must be used; this follows from the condition that p > 0. Since the rarefaction shock wave is unstable, the sign of the radical must change when D = 0. In this case, it follows from the condition D ≥ 0 that at this point dD/dh = 0 must hold.

From these conditions we find a relationship between μ and ν in parametric form

$$\mu = \frac{2h_*^2\gamma^2}{\gamma+1}\,\frac{(h_*-1)(\gamma h_* + 2 - \gamma)}{(\gamma h_* - \gamma + 1)}; \tag{10}$$

$$\nu = \mu(\gamma^2 - 1)/\gamma^2 h_* + h_*^2 - \mu - 1, \tag{11}$$

where h_* is a parameter.

The dimensionless magnitudes of the magnetic field h_0 and of the pressure p_0 in the insulators† can be calculated from Eqs. (6) and (7):

$$h_0 = 1 + \mu/4 + \nu\gamma/2(\gamma - 1); \tag{12}$$

$$p_0 = 1 + (1 + \mu - h_0^2)/\nu. \tag{13}$$

We should point out the restriction on the region of variation of μ and ν. An analysis of the dependences h(x) in the limit h, u → 1 [Eq. (4)] shows that a finite solution exists only when the following inequalities hold:

$$\gamma\nu < \mu \leqslant 2 + \gamma\nu.$$

The dependence found for μ(ν) from (10) and (11) satisfies the first inequality. The second inequality is nontrivial. It coincides with the condition for the total velocity of sound c_1

$$c_1^2 \equiv H_1^2/4\pi\rho_1 + \gamma P_1/\rho_1 \geqslant v_1^2, \tag{14}$$

which is required for stability of the solution [4]. The presence of a limiting vaporization regime, in which v_1 reaches the maximum possible value of $v_{1max} = c_1$ (analog of the Jouguet regime in combustion), is related to this restriction.

To obtain the magnitude of the velocity v_1, we must examine in greater detail the absorption of heat in zone II. It was shown above that throughout the current layer the heat flux

$$Q \simeq Q_1 = \sigma T_1^4, \tag{15}$$

†The presence of a pressure $p_0 \neq 0$ in the insulator with high magnetic fields (H ≳ 10 MOe) can lead to the appearance of an appreciable electrical conductivity. The theory being presented will then no longer be applicable.

where T_1 is the temperature of the vapor at the end of the discharge region. We shall assume that the mean free path of radiation depends only on the density and temperature $\lambda(\rho, T)$ (the assumption of a "gray" body). We shall assume a power-law dependence on temperature and density for the equation of state, mean free path, and coefficient of magnetic diffusion:

$$P/\rho = AT^n/\rho^m; \tag{16}$$

$$\lambda = \Lambda T^j/\rho^i; \tag{17}$$

$$\varkappa = K/(\rho^k T^l). \tag{18}$$

Then, from (15), we obtain

$$q_1 = \sigma\left(\frac{\nu}{A\mu}\right)^{4/n} \mu^{4m/n}\left(\frac{H_1^2}{8\pi}\right)^{4m/n-1} v_1^{8(1-m)/n-1}. \tag{19}$$

Using the smallness of q in Eq. (7), we find, retaining the leading terms in the expansion in power of u,

$$\frac{dq}{d\xi} - \frac{2\lambda v_1}{\varkappa} = \frac{\gamma}{\gamma - 1} \nu p_0 \frac{du}{d\xi}. \tag{20}$$

Here, the dependence $q(\xi)$ is described by the formula

$$q(\xi) = 2q_1 \int_0^1 \cos\theta e^{-\xi/\cos\theta} d(\cos\theta)$$

for pure absorption of photons in zone II from a Lambertian source, since, by virtue of the fact that $q_1 \ll 1$, the temperature in zone II is much lower than T_1. Transforming to the variables

$$y \equiv \frac{q}{q_1}, \quad z \equiv \frac{\gamma}{\gamma - 1} \frac{\nu p_0}{q_1} u$$

in Eq. (20) and using (16)-(18), we obtain

$$\frac{dz}{d\xi} + az^\alpha = \frac{dy}{d\xi}, \tag{21}$$

where $\alpha = i - k + (j + l)(1 - m)/n$;

$$a = \frac{q_1^{\alpha-1}}{\left(\frac{\gamma}{\gamma-1}\nu p_0\right)^{\alpha-1}} \frac{2\Lambda(\nu p_0)^{\frac{j+l}{n}}}{KA^{\frac{j+l}{n}}} \frac{v_1^{1+2\alpha}}{\mu^\alpha\left(\frac{H_1^2}{8\pi}\right)^{i-k-\frac{(j+l)m}{n}}}. \tag{22}$$

The solution $z(\xi)$ of the first-order differential equation (21) must satisfy the two boundary conditions

$$z \to [(\alpha - 1)a\xi]^{-1/(\alpha-1)} \quad \text{as} \quad \xi \to 0; \tag{23}$$

$$z(\xi) \to y(\xi) \quad \text{as} \quad \xi \to \infty, \tag{24}$$

so that we have a problem for determining the eigenvalue a. The condition (23) follows from the requirement that the solution transform smoothly to the solution in zone I in the limit $h \to h_0$. The condition (24) follows from the fact that the heat flux is responsible for the initial heating of the insulator. Substituting the eigenvalue a obtained after the solution of the problem into (22), we can calculate the velocity v_1.

Let us examine a specific, quite typical, example. For Plexiglas, using Saha equation with multiple ionization [5], the mean free path of radiation with multiple ionization [5], and the electrical conductivity of a Lorentzian electron gas [6], in the region of temperatures 3-30 eV and densities 10^{-3}-10^{-5} g/cm^3, we can obtain the following values of the constant in formulas (16)-(18):

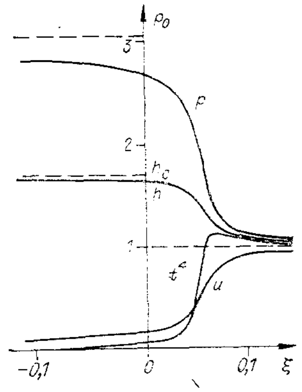

Fig. 1

$$A = 0{,}17,\ m = 1/16,\ n = 19/16,\ \gamma = 4/3,$$
$$\Lambda = 2\cdot 10^{-9},\ i = 1{,}857,\ j = 2{,}143,$$
$$K = 0{,}17,\ k = 0{,}143,\ l = 0{,}857. \tag{25}$$

We are using the system of unit g, cm, μsec; the temperature is measured in eV. In these units, $\sigma = 1.03 \cdot 10^{-6}$.

The numerical solution of (21) with the boundary conditions (22)-(23) gives $a \simeq 2$ and, from (22),

$$v_1 = \frac{5.5 p_0^{0{,}061} \mu^{0{,}541}}{\nu^{0{,}345}} (H_1^2/8\pi)^{0{,}156}. \tag{26}$$

We note that the dependence of the solution obtained on the magnitude of the mean free path is very weak (the velocity of the vapor $v_1 \sim \lambda^{0.04}$). For this reason, it is natural to expect that the inaccuracy inherent in the gray-body approximation likewise has virtually no effect on the result.

The formula (26), combined with formulas (10)-(13), gives a relationship between the velocity of the vapor v_1 and the fraction of the current branching off into the discharge $(1 - 1/h_0)$ as a function of the magnitude of the magnetic field H_0. This relationship is the boundary condition in the complete MHD problem.

The limiting regime of vaporization, corresponding to the equality in formula (14), corresponds to the following values of the parameters:

$$\mu = 2{,}12,\quad \nu = 0{,}091,\quad h_0 = 1{,}71\quad p_0 = 3{,}04,$$
$$v_{1\,\max} = 17\,(H_0^2/8\pi)^{0{,}156}, \tag{27}$$

and the limiting power transferred through the surface of the insulator equals

$$v_1 H_1 H_0/4\pi = 20\,(H_0^2/8\pi)^{1.156}.$$

The quantity q_1 for the example for the example under examination in the limiting regime

$$q_1 = 0.099\,(H_0^2/8\pi)^{0.041}$$

is virtually independent of H_0 and for $H_0 < 10$ MOe is of the order of $q_1 \lesssim 0.1 \ll 1$, which confirms our assumption that the radiation energy flux is small.

To illustrate the solution obtained, the dependences of the basic magnetohydrodynamic quantities on the optical thickness $\xi[t^4 \equiv (T/T_1)^4]$ are illustrated in the figure for the limiting regime.

When the velocity v_1 drops below the limiting value, the fraction of the current branched off also decreases. For example, when $v_1 = 0.61 \times v_{1}\max$, the fraction of the branched current equals $(1 - 1/h_0) = 0.11$ ($h_0 = 1.128$, $\mu = 0.31$, $\nu = 0.026$). If, however, the velocity of the boundary of the vapor leaving the insulator in the complete MHD problem exceeds $v_1\max$, then, since the velocity of the vapor near the insulator remains equal to $v_1\max$, a rarefaction wave forms between the insulator and the boundary [2].

We note that we have also obtained the numerical solution of the problem within the framework of the diffusion approximation for the heat transfer. In spite of the formal inapplicability of this approximation, the results (as often occurs with the diffusion approximation) turn out to be quite close to the ones presented above. In particular, the limiting velocity v_{1max} depends in almost exactly the same manner on H_0 as in (27); in addition, the difference for H_0 = 1 MOe is only of the order of about 10%.

As we indicated above, the theory presented here is applicable only for high magnetic fields $H \sim 10^7$ Oe (see footnote above concerning the pressure in the insulator). Otherwise, the theory should be applicable in all cases when the equation of state, the mean free path, and the conductivity of the insulator vapor can be described by power-law formulas (16)-(18) and when the vaporization process is stationary. The latter condition presumes quite slow variation of the magnetic field over a time $\sim\lambda/v_1$, during which the insulator particles fly away over a distance of the order of the mean free path of the photons, in order that the intensity of the irradiation of the insulator surface likewise vary slowly and correspond to the intensity of blackbody radiation of the vapor σT^4_1.

The specific values of the parameters (25), as indicated above, were selected for $H_8C_5 \cdot O_2$* with magnetic fields of $H \sim 10^5-10^6$ Oe and velocities $v_1 \sim 10^6-10^7$ cm/sec in mind, in order to investigate in the theory the process of vaporization of the insulator in fields exceeding [3] $H \sim 10^4$ Oe [1, 2]. We note that although the parameters (25) were not specially calculated for fields of $H \sim 10^4$ Oe, used in the experiments in [1], the results calculated with these parameters for the velocity v_1 and the velocity of the shock wave in the magnetic shock tube agree reasonably well (taking into account the possible appreciable difference between the intensity of irradiation of the insulator and the blackbody radiation of the vapor due to the small optical thicknesses in [1]) with the experimental values in order of magnitude and give approximately the same dependence of the velocities on the magnetic field: When H varies by an order of magnitude, the velocity varies in the experiment by a factor of 2-3.

LITERATURE CITED

1. J. Keck, "Current speed in a magnetic annular shock tube," Phys. Fluids, 7, No. 11, Part 2 (1964).
2. J. B. Workman, "Insulator ablation in a magnetic piston shock tube," Phys. Fluids, 8, No. 12 (1965).
3. W. L. Baker, M. C. Clark, et al., "Electromagnetic-implosion generation of pulsed high-energy-density plasma," J. Appl. Phys., 49, No. 9 (1978).
4. L. D. Landau and E. M. Lifshitz, Mechanics of Continuous Media [in Russian], Gostekhizdat, Moscow (1954).
5. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High Temperature Hydrodynamic Phenomena, Academic Press.
6. E. M. Lifshitz and L. P. Pitaevskii, Physical Kinetics [in Russian], Nauka, Moscow (1979).

*There is a very weak dependence of the parameter q_i on the atomic weight A_i ($q_i \sim A_{ins}^{0.1}$) with very similar ionization potentials for all the materials, thus permitting the application of numerical relationship (26) for all other insulating materials.